**Homework 2 Solutions**

**Problem 1.** As part of your job as a the most brilliant physicist ever, you derive the following equation for how waves propagate through a certain novel material, where α = 32N, and β = 2kg/m.

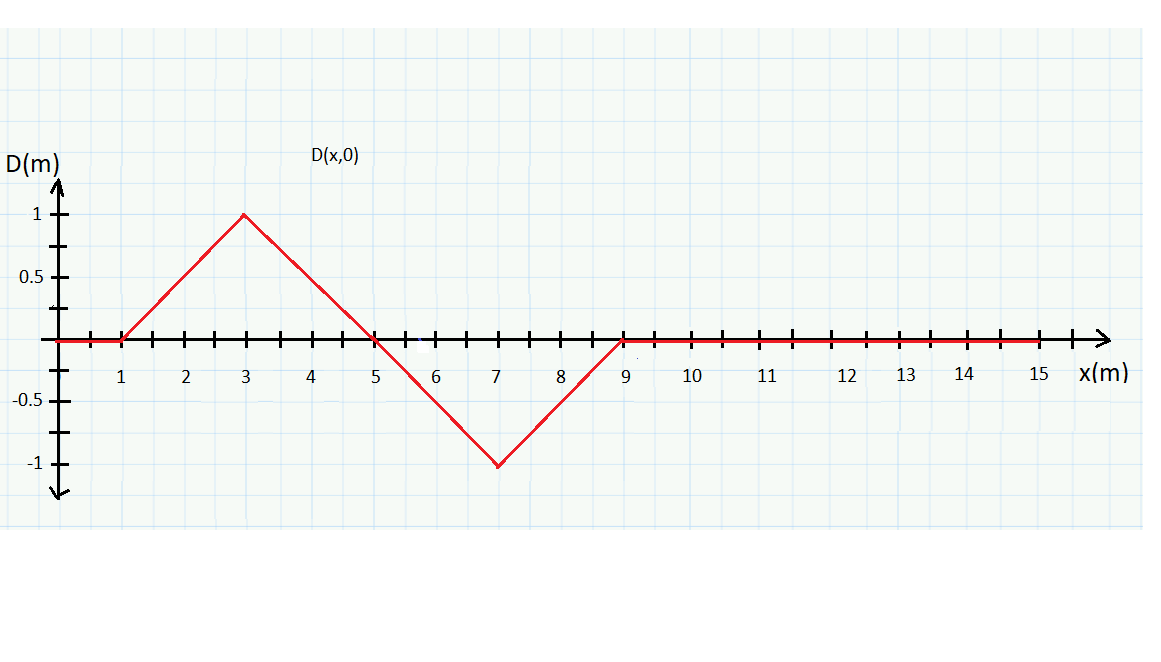


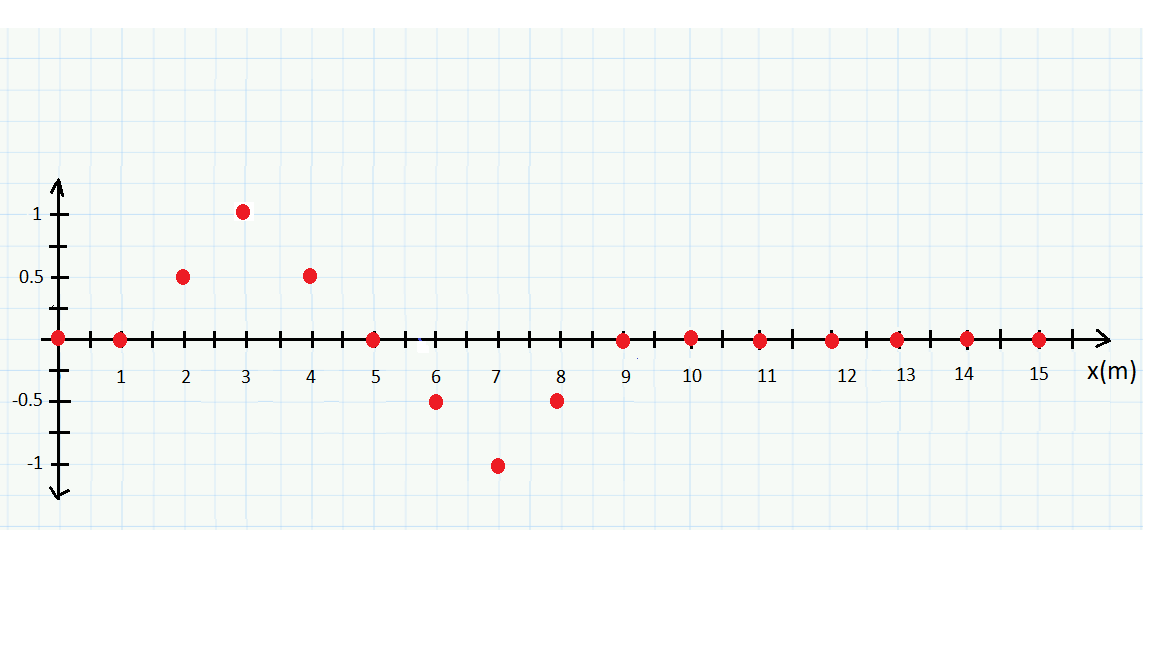
(a) With what velocity will a wave propagate through this material?

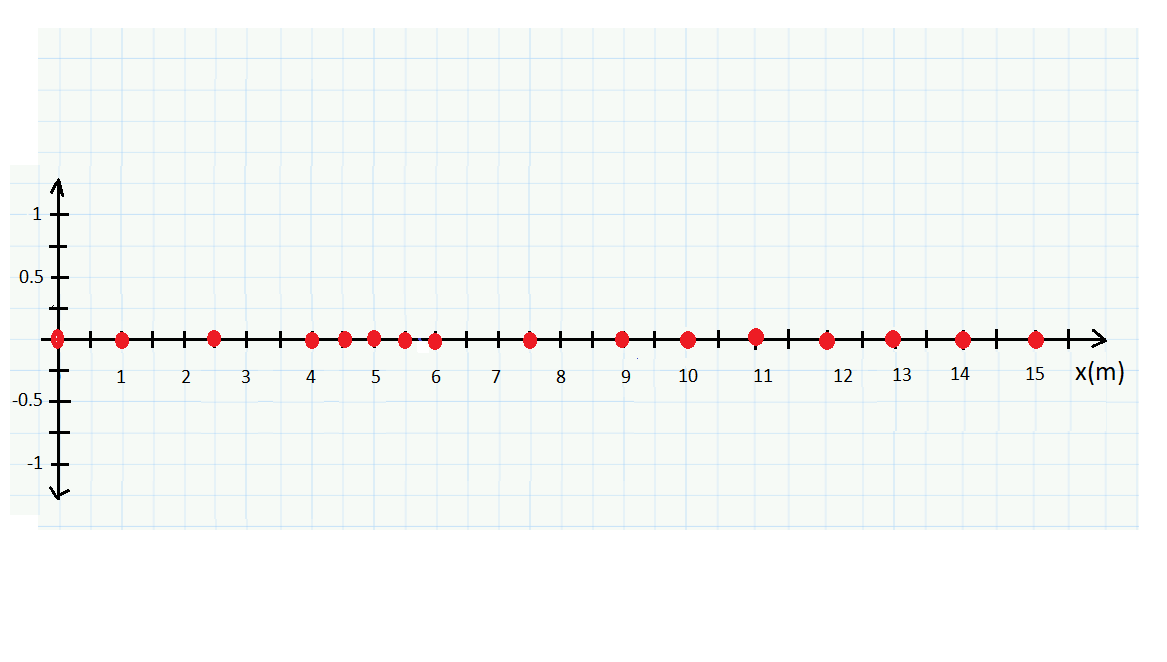
Well,



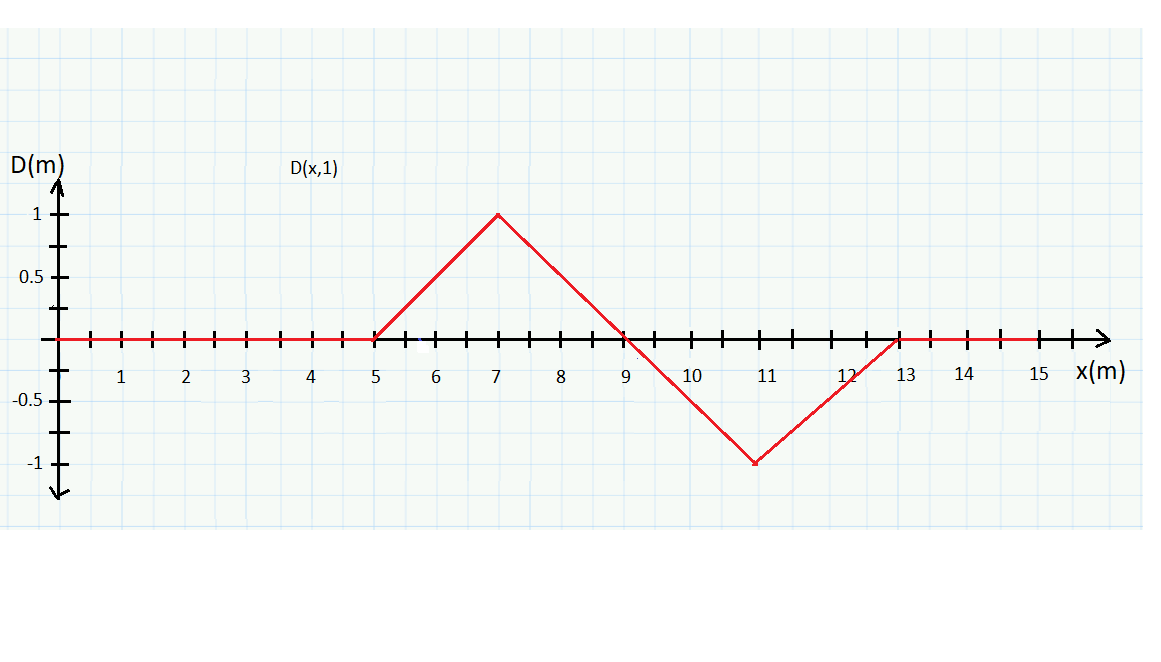
(b) Suppose the particles in the medium were all initially at integer positions, but then are displaced according to the following graph D(x,0). Draw the actual position of the particles if this is a transverse wave. And then if it’s a longitudinal wave.

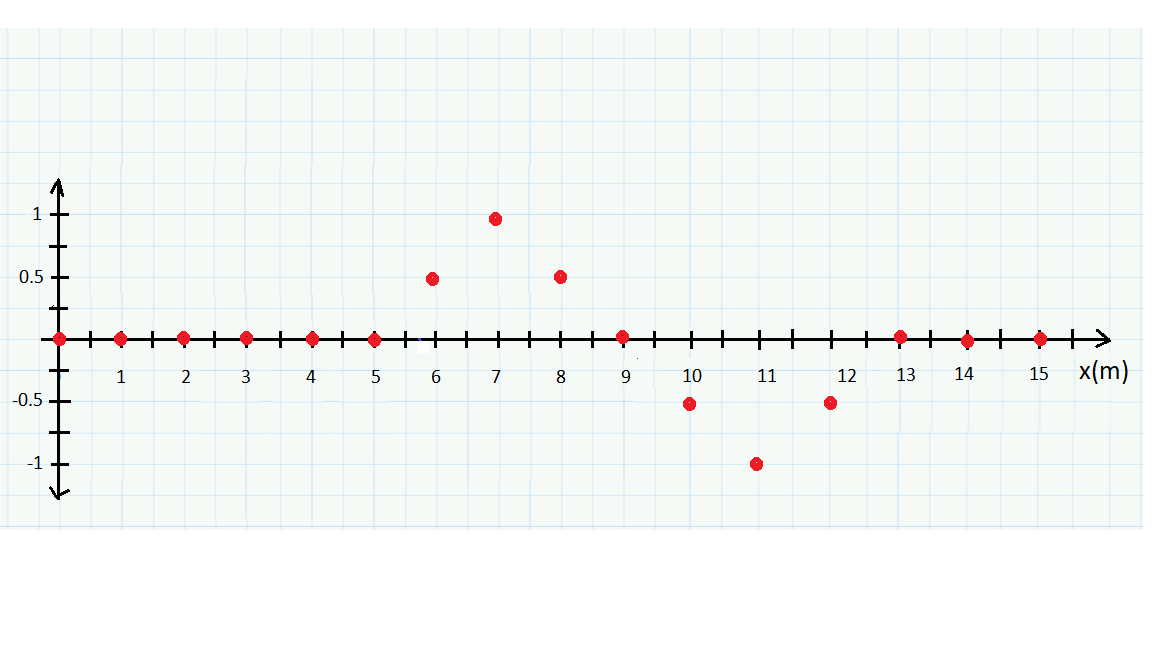


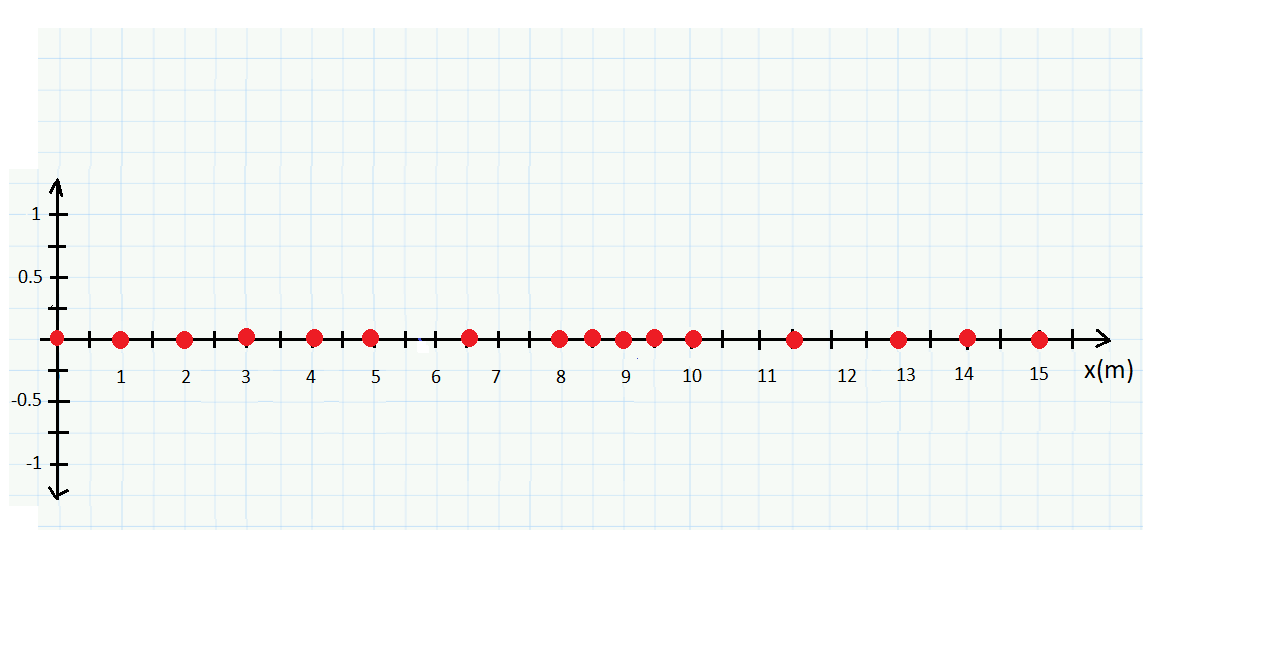




(c) What will D(x,1) look like? And draw the particles’ positions at t = 1, for both the transverse and longitudinal cases:



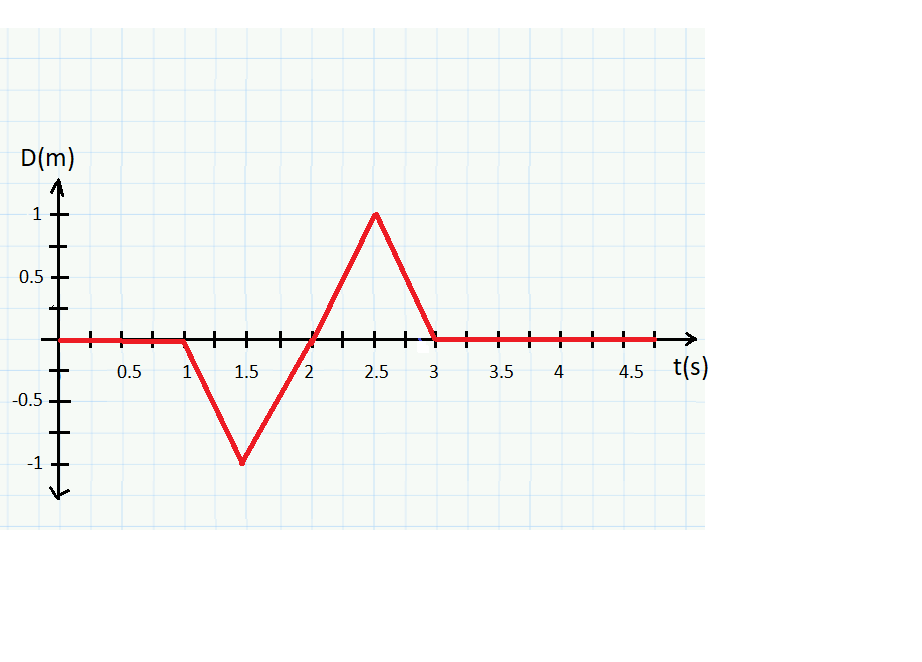




(d) What is the displacement and velocity of the x = 10m particle at time t = 1s? What direction is it moving in (for transverse and longitudinal cases)

So D(10,1) = -0.5. So displacement is 0.5m down (transverse) or to the left (longitudinal) and v(10,1) = -slope∙vwave = -(-1/2)∙4m/s = 2m/s. It’s therefore moving 2m/s up (transverse) or to the right (longitudinal).

(e) Draw a history graph of the point x = 13m, i.e. D(13,t). Between what times is it moving down (transverse) left (longitudinal)? Between what times is it moving up (transverse) right (longitudinal)?



It’s moving down/left between t = (1s,1.5s) and (2.5s,3s), and up/right between t = (1.5s,2.5s)

**Problem 2.** Suppose a wave is traveling through a string at velocity v = -20m/s, and that the initial displacement function is , measured in m.

(a) What is D(x,t)? and what is a general expression for the velocity of any point at any time?

Duh.



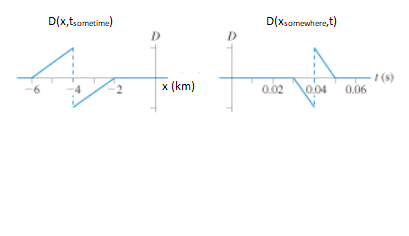
(b) What is the displacement and velocity of the point x = -38m at t = 2s? What direction is the direction of its displacement in the transverse/longitudinal cases? What is the direction of its velocity in the transverse/longitudinal cases?

Well,



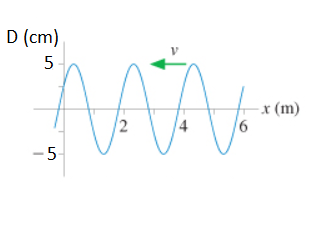
So its 2m up/right and moving 16m/s down/left.

**Problem 3.** When Alderaan exploded, it sent a disturbance through the force. After Obi-Wan Kenobi felt it, it just kept going out into space, eventually making its way to Earth. Physicists here have recently developed a force-sensitive seismograph and it captured the following pictures of the disturbance, plotted below in the form of graphs D(x,tsometime), and D(xsomewhere,t), where xsomewhere and tsometime are definite points in space, time but otherwise unspecified. With what speed and in what direction is the force wave propagating?



From snapshot graph, we can see wave is Δx = -2km – (-6km) = 4km long. And from history graph we can see that it passes through the point x = 2km in Δt = 0.05s – 0.03s = 0.02s. So the wave velocity is v = Δx/Δt = 4km/0.02s = 200km/s. Moreover, it is going to the right because we can see that the point xsomewhere first goes down and then up. And this is what would happen if the wave is traveling right, since the downward-sloped portion of the graph would get to that point first.

**Problem 4.** A mountain climber (m = 70kg) is hanging from a rope. The rope has a mass m = 2kg, and length 20m. When he loses his footing, he inadvertantly shakes the rope, setting up a sinusoidal wave in the rope, depicted below (origin of axis is at the point where rope is anchored, whereas climber is at coordinate x = 20m). At some arbitrary time we will call t = 0, the rope looks like this:



(a) Write down the equaion for D(x,0).

So D(x,0) = Asin(kx+φ0). A = 5cm clearly. k = 2π/λ = 2π/2 = π. And to get the phase constant, we could use φ0 = (2π/λ)Δx0, where Δx0 is the displacement of the wave from the origin, but this difficult due to the lack of scale on the graph. So rather we can solve for φ0 knowing that D(0,0) = 2.5. So,2.5 = 5sin(π∙0 + φ0) → 2.5 = 5sin(φ0) → φ0 = π/6 (other possible solution 5π/6 doesn’t work here). So D(x,0) = 5sin(πx + π/6).

(b) Write down the equation for D(x,t).

We need v, which is:



Now since the wave is traveling to the left, v = -83m/s. Therefore,



(c) What is the frequency of oscillation of the string?

Since we have ω = 83π, the frequency follows as f = ω/2π = 83/2 = 41.5Hz.

(b) What is the maximum displacement of a point on the string?

That would be A, which is 5cm.

(c) What is the maximum speed of a point on the string?

That’s



And so the max velocity would be:



(d) What is the maximum acceleration of a point on the string?

And max acceleration is:



Which makes the max acceleration:



(e) If an ant (m = 2mg) on the rope can exert a maximum gripping force of 0.1mN, will it be able to stay on the rope?

Well, as long as the max acceleration the ant can potentiate is larger than the acceleraion the rope would induce, then yes. So we have to compare aant = Fant/m = 0.1×10-3/2×10-6 = 0.05×1000 = 50m/s2 to arope = 34m/s2. So it appears the ant will be able to hold on.

(f) Calculate the linear energy density along the length of this rope.

Energy density is:



(g) What is the energy in one of its wavelengths?

This is the energy density times the length, which is:



(h) What is the intensity of the wave along the rope?

Intensity is I = uv, which is:



(i) What is the rate at which energy is being delivered to the anchor bolt? This could be a consideration vis a vis whether it comes lose. Just sayin.

This is the power, and in 1D, power is the same as intensity, so 714W are being delivered to the anchor point.

**Problem 5.** You’re fed up with school and have decided to make an attempt on the SLO-to-Japan single person kayak speed record. You’re taking a break from rowing one fine afternoon and notice waves passing by. Your kayak is bobing up and down 20 times per minute. A rubber ducky you have tied to your kayak (for good luck) bobs up and down with the waves, and a gps sensor records its maximum speed as 0.70m/s. Finally, you notice that it takes a given wave crest about 1s to pass your 5m long kayak. Using the rubber duck as the origin of your coordinate system, and starting time from when it is at its nadir, write an equation for D(x,t) = Asin(kx-ωt+φ0), assuming the waves are sinusoidal of course, and that the velocity of the wave is ‘positive’.

So simple!

So f = 20/60 = 0.33Hz → ω = 2πf = 2π/3 rad/s = 2.1rad/s.

The wave velocity is 5m/1s = 5m/s.

And so the wavelength of the wave is λ = v/f = 5/0.33 = 15m → k = 2π/λ = 0.42m-1.

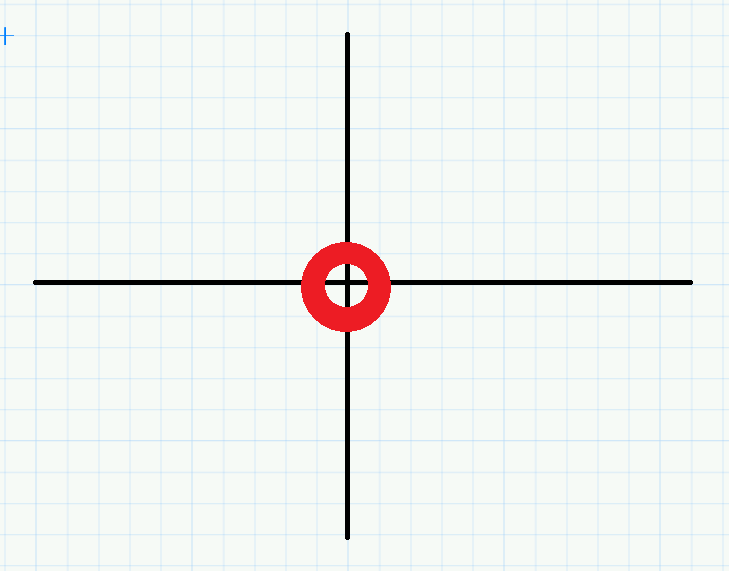
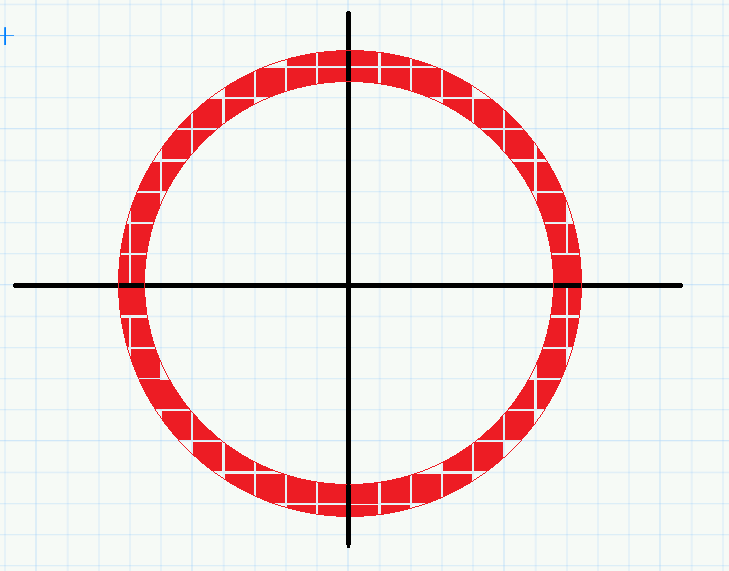
And from the rubber ducky we see that vmax = 0.70 → Aω = 0.70 → A = 0.70/ω = 0.70/(2π/3) = 0.33m. Finally, since the ducky is at the bottom of the cycle, the phase constant is φ0 = (2π/λ)(λ/4) = π/2.

So D(x,t) = 0.33sin(0.42x – 2.1t + π/2)

**Problem 6**. A bomb explodes in foam-like substance being examined for its protective properties. Its density is ρ = 0.2kg/m3, and its bulk modulus is B = 10N/m. Suppose at time t = 0, the displacement of the foam’s constituent particles is described by



In other words, D = 1cm inside the colored region, and 0 everywhere else.

(a) What is the wave pulse’s velocity?

Wave velocity is v = √(B/ρ) = √10/0.2) ≈ 7m/s.

(b) Depict D(r,1) on the graph below. What is its rough amplitude?

After 1s, the pulse will have spread outwards roughly 7m therefore (to a radius of 8cm). It’s amplitude would have decayed to roughly A/r = A/8 = 1cm/8 = 0.12cm. And it will look like this above:

**Problem 7.** Consider the same foamy substance, and suppose that the initial 3D pulse is described by the function: , where D is measured in cm.

(a) What is the expression for D(r,t), and v(r,t)?

Well, we have D(r,t) = f(r-vt)/r. Now D(r,0) = f(r)/r → f(r) = D(r,0)r = 3rexp(-2r2). And so:



(b) Consider a point at radius r = 6m. What is its displacement and velocity at t = 1s? Describe the Direction of the displacement if it’s a transverse/longitudinal wave, and direction of the velocity as well.

So then we just have to fill in r = 6 and t = 1. So that’s:



So D is downward tangent to the surface/towards origin if transverse/longitudinal, and velocity is upward tangent to the surface/away from origin if transverse/longitudinal.

**Problem 8.** Now suppose that the aforementioned explosion sets up a sinusoidal wave described by the following equation, where D is measured in cm again.



(a) What is the wave velocity?

v = ω/k = 60/3 = 20m/s.

(b) What is wavelength?

Since k = 2π/λ, λ = 2π/k = 2π/3 = 2.1m.

(c) What is the frequency of oscillation?

Since ω = 2πf, f = ω/2π = 60/2π = 9.6Hz.

(a) What is the amplitude of oscillation at the radius r = 10m?

Amplitude is coefficient of sin function, whch would be 5/10 = 0.5cm at that point.

(b) What is the max velocity of the medium at radius r = 10m?

The velocity at that point is: v = dD/dt. And so max would be: (5/r)∙60 = (5/10)(60) = 30cm/s.

(c) What is max acceleration of the mediumm at radius r = 10m?

The acceleration at that point is a = d2D/dt2. And so max would be (5/r)(602) = (5/10)(602) = 1800cm/s2.

(d) What is the energy density of this wave (as a function of r)?

Energy density is u = (1/2)ρ(Aω/r)2. So,



(e) What is the energy in one wavelength of this wave? Note that we must integrate the energy density over the volume comprised by one wavelength, i.e. Eλ = ∫udV, where dV is differential volume. It could be helpful to know that in radial coordinates, dV = 4πr2dr.

If we integrate then,



(f) What is the intensity of this wave (as a function of r)?

That’s I = uv, which is:



(g) What is the rate at which energy is passing through the foam?

This the power, P = IA, which is:



**Problem 9.** The B-70 Valkrie, a Mach 3 capable bomber, might have been the loudest jet on takeoff (or might not, trying to remember what I read in 6th grade). Suppose that 15km away from the engine, the volume was 60dB.

(a) What is the intensity of the sound wave at this point?

Intensity is given by:



(b) What was the power of the sound waves coming from the engine?

Power is IA, which is:



(c) How close to the engine could you get before damage was done to your ears?

Comparing two different points,



(d) From what maximum distance could you still hear the engine (this is a ridiculously huge number, but in reality, the distance would be far far far far less since energy does get dissipated by the viscocity of the air medium)?

This would be given by:



Yep. That’s a lot, cause it’s about from here to China or something.